Multi-Depot Truck and Trailer Routing Problem with Multiple Time Windows

A case study of the historic center of Querétaro

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***Abstract*— The objective of this work is to solve an extension of the Truck and Trailer Routing Problem where several depots and customers with multiple time windows are considered. This extension is motivated by a real-life problem arising in the historic center of Querétaro (México) where several companies need to deliver different types of products to customers. Because of access limitations, trucks cannot park at the customer location, instead nearby bays are used as park places. In these bays, products are transferred from the truck to a dolly held in the truck. Having transferred the product to the dolly, this is manually driven to the customers by the truck driver, and/or by one or two support operators, depending on the specific company. Customers are usually visited at morning time window, but some of them can be also attended at afternoon time window. State-of-the-art works do not consider neither multiple depots nor multiple time windows. Moreover, truck access to bays must be coordinated to not overlap park times, as reachable customers from these bays are served by several companies. This possess a great difficulty when planning the routes of all operating companies. The problem is formulated as a mixed-integer linear programming model where the companies' benefits are maximized. A route-based representation on a directed graph is used in order to efficiently solve real-sized instances. Customer clustering and route enumeration methods are devised for defining the candidate routes to be evaluated by the model. The daily delivery of fruits juice is analyzed and the results show that service times from the park places to the customers are three times higher than travel times among bays.**

Keywords*— *Truck and trailer problem; Multi-depot; Multiple time windows; Mixed-integer linear programing model; Customer clustering; Route enumeration; Historic center of Querétaro

I. INTRODUCTION

In emerging cities, people are buying more and more frequently consumed products in nanostores [1] (i.e., convenience stores, family stores, among others). The increase demand of such products is requiring more and more trips and the use of bigger vehicles to deliver the product to these stores. However, the locations of the stores are not usually accessible for such vehicles because nearby streets are too narrow to go into, or the streets' pavements may be heavily deteriorated if vehicles pass by, among other reasons. Therefore, vehicles are parked in available park places within an allowable distance from the stores where the demanded product is transferred to smaller vehicles that eventually delivers it to stores.

This real-life situation is known in the scientific literature as The Truck and Trailer Routing Problem (TTRP) [9]. In this problem, a fleet of vehicles composed by combinations of trucks and trailers need to deliver the product to customers, some of them situated in zones with accessibility limitations. To overcome these limitations, the trucks depart from the depot with an attached trailer that carries the demanded product. The vehicle parks at one or several places where part or the total of the product carried in the trailer is transferred to the truck, which eventually delivers the product to the customer near the park place.

Some extensions to the TTRP have been study. Among them, the TTRP with Time Windows [2,3,5-8,12-14,17,19,20]. Leaving aside [2, 8], no other works allow customers to be attended by more than one vehicle. This feature is compulsory in scenarios, for instance, with scant park places, and narrower time windows. In those scenarios, the delivery company may decide to split the demanded product of certain customers into several trucks. Another scenario is when several companies operates within the same customers' zone, which is our case.

The objective of this work is to solve an extension of the TTRP incorporating multiple depots as well as multiple time windows. Each depot represents the departure and ending of trucks' routes from a given company, whereas each time window stands for a given time interval of the day where customers can be attended. To the best of our knowledge, none of these features has previously been studied in the literature. This TTRP extension is formulated as a mixed-integer linear problem (MILP) defining the vehicles coordination constraints of [2] in the park places instead of in the customers' locations. A route-based representation, which differs from the ones encountered in the literature [5,12,14], is also proposed.

The remainder of the paper is organized as follows. First, a survey on relevant studies on TTRP with time windows is conducted. Second, the problem statement is defined. Third, the underlying graph representation of the problem is developed. Fourth, the mathematical formulation of the MILP is presented. Fifth, preliminary results are reported on the planning of fruits juice delivery. The article finishes with some conclusions and directions for further research.

II. Related works

The TTRP with time windows (TTRPTW) is first introduced by Semet & Taillard [17]. In that work, a real-life problem of a Swiss company is presented. The company delivers goods to grocery stores located in Switzerland. To perform the deliveries, the company owns a heterogeneous fleet of vehicles, each of them characterized by a capacity and a cost per kilometer traveled. Each vehicle consists of a truck that can pull a subset of the available trailers, and each customer must be visited by a compatible vehicle within a given time window. A metaheuristic based on a tabu search, along with some strategies to speed up computing times, is proposed to solve that problem.

Zitz [20] extends Semet & Taillard‘s problem to separate park places from customers. In this extension, the customers at which trucks with attached trailers can directly arrived are no longer considered as feasible parking places to serve nearby customers with accessibility limitations. In addition, load restrictions are also considered to determine feasible combinations of trucks and trailers. A 36 instance-benchmark based on benchmark instances from the capacitated vehicle routing problem with time windows (CVRPTW) is built. An attribute-based hill climber metaheuristic is used for solving the generated instances.

Lin et al. [12] generate a 54 instance-benchmark for the TTRPTW based also on the benchmark instances from the CVRPTW. A simulated annealing metaheuristic, inspired on authors' previous work in [11], is used to solve the generated instances.

Derigs et al. [5] study the impact of freight transfer between the trailer and the truck. The authors point out that if that transfer is forbidden, the demands of all customers served on a subtour must be loaded onto the truck at the depot and, on the other hand, only the demands of vehicle customers visited in the main tour can be loaded onto the trailer. The authors present a hybrid algorithm that combines local search and large neighborhood search moves guided by two simple metaheuristic control strategies. Computational experiments provided on Lin's benchmark instances show that the proposed method outperforms the algorithm defined in [12].

Drexl [6] presents another extension to the TTRPTW where the product is transferred in the opposite way (i.e. from the truck to the trailer. This extension is motivated by a real-life problem related to the milk collection in Bavaria region of Germany. Additionally, fixed costs for using the vehicles are considered. The author proposes two MILP formulations based on arc-flows, and path-flows, respectively. A branch-and-price (B&P) algorithm is developed for the path-flow formulation, as well as some heuristic variants for the arc-flow formulation. Computational experiments are run for randomly generated instances structured to resemble real-life problem scenarios. The computational results show that the generated instances can be solved in short computing times with high solution quality using the B&P algorithm on the path-flow formulation.

Drexl [7] proposes different branch-and-cut strategies for the arc-flow formulation of a similar problem as the one presented in [6], by studying several families of valid inequalities. Unlike [6], this problem can determine the optimal trailers-to-trucks assignment by allowing load transfers among vehicles.

Batsyn & Ponomarenko [2] study the product delivery of a big retail Company. A MILP model is developed including hard and soft time windows. The former establishes the opening and closing times of stores, whereas the latter set the preferred times for delivering the product. Another new feature incorporated in the MILP is that the product delivery may be split among several vehicles. Therefore, coordination delivery constraints on customers are imposed such that only one truck attends a customer within a specific time interval. A multi-start greedy heuristic is proposed to solve de MILP.

Belenguer et al. [3] solve a single truck and trailer routing problem by a similar approach as in [7]. In this problem, a single truck towing a detachable trailer has to serve a given set of customers only accessible by truck. Appropriate locations to park the trailer and transfer load from the trailer to the truck have to be selected.

Mirmohammadsadeghi & Ahmed [13] incorporate stochasticity on the demand and time windows of the customers. A failure probability to attend some customers is determined, and the vehicle in route may be allowed to return to those customers if possible. Otherwise, a single trip is performed. The 54 instances from Lin's benchmark are adapted to account for the stochastic issues, and a memetic algorithm with various crossovers, mutations and local search approaches is applied.

Grechikhin [8] proposes another approach to solve Batsyn & Ponomarenko's model [2]. This approach consists of a greedy heuristic that obtains an initial solution. After the greedy heuristic, the obtained solution is reconstructed by an iterative local search heuristic. The author shows that his approach outperforms the one in [2] when applied to a set of real-world instances.

Parragh & Cordeau [14] develop a branch-and-price (B&B) algorithm for the basic TTRPTW. Unlike other similar approaches, this one includes an adaptive large neighborhood search (ALNS) to populate the column pool in the B&B scheme. The incorporation of the ALNS allows achieving highly competitive results when compared to best-known results on Lin's benchmark instances.

Finally, Rothenbächer et al. [16] introduce a branch-and-price-and-cut algorithm for the TTRPTW of [6] with two additional features: 1) a two-day planning horizon during which each customer can be visited once or twice, and 2) load transfer times that depend on the amount moved from a truck to the associated trailer. Computational experiments performed on single-day instances without load transfer times show that this algorithm is faster than the ones of [7,9].

Recent surveys covering other TTRP extensions can be found in Li et al. [10], Cuda et al. [4], Prodhon and Prins [15] and Torres-Pérez et al. [19].

Table I shows the features of our problem that are incorporated into the mentioned works. One may observe that no work considers multiple depots and multiple time windows. Moreover, only the works of Batsyn & Ponomarenko [2] and Grechikhin [8] consider to attend the same customer with several vehicles.

TABLE I. FEATURES INCORPORATED IN THE RELATED WORKS

Work	FEATURES				
	Multiple Depots	***Vehicle Fleet***	***Customer Delivery***	***Time Windows***	***Route Duration***
Semet & Taillard [17]	No	Homogeneous	1 vehicle Only	Single	Unlimited
Zitz [20]					Limited
Lin et al. [12]					Unlimited
Drexl [6,7]		Heterogeneous			Limited
Derigs et al. [5]		Homogeneous			Unlimited
Batsyn & Ponomarenko [2]		Heterogeneous	Several Vehicles		Limited
Belenguer et al. [3]		Homogeneous	1 vehicle Only		Unlimited
Mirmohammadsadeg & Ahmed [13]					Unlimited
Grechikhin [8]		Heterogeneous	Several Vehicles		Limited
Parragraph & Cordeau [14]		Homogeneous	1 vehicle only		Unlimited
Rothenbächer et al. [16]	No	Heterogeneous		Single	Limited
This work	Yes	Heterogeneous	Several Vehicles	Multiple	Limited

Complementary, Table II shows the methodological aspects. It is clear that metaheuristic approaches are preferred against exact ones when solving the TTRPTW. In fact, no exact approach is capable of solving large-sized networks. However, using a route-based formulation may help to improve the performance as shown in Parragh & Cordeau [14]. Also, important to observe is that only two works report results using real data apart from us.

TABLE II. METHODOLOGICAL ASPECTS OF THE RELATED WORKS

Work	***Approach Type***	***Model Formulation***	***Data type***	***Data size***
Semet & Taillard [17]	Metaheuristic	Arc-based	Real	Small
Zitz [20]			Experimental	Medium
Lin et al. [12]			Experimental	Large
Drexl [6]	Exact	Route-based	Experimental	Small
Drexl [7]		Arc-based		
Derigs et al. [5]	Metaheuristic			Large
Batsyn & Ponomarenko [2]			Real	
Belenguer et al. [3]	Exact		Experimental	Medium
Mirmohammadsadeg & Ahmed [13]	Metaheuristic			Large
Grechikhin [8]		Arc-based	Real	
Parragraph & Cordeau [14]	Exact	Route-based	Experimental	Medium
Rothenbächer et al. [16]	Exact			Small
This work	Exact		Real	Large

Small: < 50 customers, Medium: 50-100 customers, Large: >100 customers

III. PROBLEM STATEMENT

Consider a set of customers representing different types of nanostores situated in an urban zone with limited access. These customers demand specific products to different companies whose depots are located on the periphery of the city. Each company operates at least with one type of truck differing in terms of product capacity, dimensions, and travel speed with respect to the other companies' trucks.

Customers are usually located within narrow streets in which trucks have no access, so nearby bays are used as park places. In these bays, truck operators transfer the product from the truck to a dolly that is then manually driven to the customers. This process is repeated, at least, as many times as the number of customers to be served from that bay because of the dolly capacity and weigh limitations. Therefore, dollies are not driven among customers, only from bays to customers and vice versa.

Time access restrictions also exist in both bays and customers' locations. In some periods of the day, trucks may be forbidden to enter bays for many reasons (for instance, to decrease traffic congestion). Regarding to customers, there are some breaks for having meal, for instance, during which stores are closed.

The aim of the problem is to determine for each company: 1) which bays are used, 2) which customers are served from each used bay, 3) the visiting sequence of the bays/customers, 4) how many trucks are needed, in such a way that a certain amount of product is delivered to certain customers maximizing the companies' benefits.

The problem will be represented as a mixed-integer linear programming model with the following assumptions:

- The customers to which companies must serve the products and their quantities are deterministic. Therefore, once a truck starts its route, no new product demands can be attended.
- Trucks start and end their routes at their corresponding depots within a specific time window.
- Trucks of different companies do not share products.
- Each company only attends a customer once.
- Several trucks, either from the same or different company, can use the same bay, but park times must be coordinated

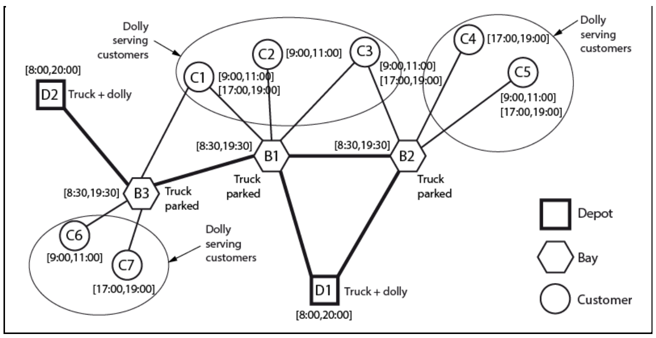
such that one truck departs from the bay before parking the other truck.

- A truck can wait as long as necessary before entering the bay if another truck occupies it.

IV. GRAPH REPRESENTATION OF THE PROBLEM

Fig. 1 depicts an illustrative example of the problem stated in the previous section.

Fig. 1. Example with 2 companies, 3 bays, and 8 customers.



This example consists of two companies located at depots labeled as D1 and D2, from which trucks must start/end their routes. Three bays labeled as B1, B2 and B3 can be used as park places and for transfer the product to the dolls to serve the seven customers tagged as C1, C2, C3, C4, C5, C6 and C7. Thicker lines represent bi-directional links connecting pairs of bays as well as depots with bays, whereas thinner lines draw bi-directional links connecting bays to customers. Finally, the brackets near to bays and customers contain the earliest and latest arrival times in hours. Some customers have two brackets allowing dollies to deliver the demanded products within two distinct time windows. Similarly, the brackets near to depots denote their earliest departure and latest arrival times in hours.

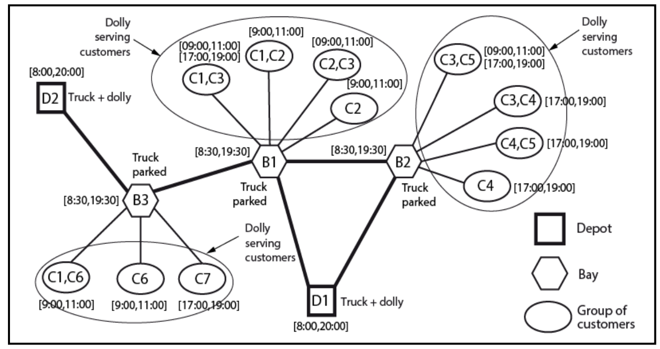
Let us suppose that each customer requires a total service time of 45 minutes (including the time spending on loading/unloading the product and on manually driving the dolly to the customer), no matter the type of product delivered. Since the bandwidth of all the customers' time windows is equal to 2 hours and the time windows of bays and depots are not restrictive, a maximum of two customers can be delivered product within the same window.

To carry out customers' clustering procedure, the following rules must be satisfied:

- Rule 1. Customers on the same group must belong to the same time window.
- Rule 2. Join as much as many customers within each time window.
- Rule 3: This is an exception to Rule 2. Create smaller groups with customers reachable from only one bay at single time window.

Looking at previous Fig. 1, one may see that customers C2, C4, C6 and C7 can only be served from bays B1, B2 and B3, respectively, and within a single time window. Therefore, Rule 3 must be applied. The rest of customers are grouped according to Rules 1-2. The resulting customers' clustering is shown on Fig. 2.

Fig. 2. Resulting customers' clustering on the example.



From the graph representation shown in Fig. 2, the candidate routes to be used by trucks are determined. To this end, some rules are also applied as follows:

- Rule 4. Each bay is only visited once
- Rule 5. Respect the time windows of the customers' groups associated with each bay
- Rule 6. Serve as much customers as possible

Table III contains the 22 possible routes for each company following the mentioned rules. Observe that, several routes have the same scheduled bays; however, the scheduling of customers is different. In addition, some of them contained less number of attended customers. This happens because some customers were grouped alone following Rule 3.

TABLE III. CANDIDATE ROUTES FOR EACH COMPANY ON THE EXAMPLE

Route	Visited Bays	Attended Customers
1	B1 [09:00-11:00] B2 [17:00-19:00]	C1, C2, C3, C4
2		C1, C2, C3, C5
3		C1, C2, C4, C5
4		C1, C3, C4, C5
5		C2, C3, C4, C5
6		C2, C1, C3
7		C2, C3, C5
8		C2, C3, C4
9		C2, C4, C5
10	B1 [09:00-11:00] B3 [17:00-19:00]	C1, C2, C7
11		C1, C3, C7
12		C2, C3, C7
13		C2, C7
14	B2 [09:00-11:00] B3 [17:00-19:00]	C3, C5, C7

15		C1, C6, C3, C4
16		C1, C6, C3, C5
17		C1, C6, C4, C5
18	B3 [09:00-11:00]	C1, C6, C4
19	B2 [17:00-19:00]	C6, C3, C4
20		C6, C3, C5
21		C6, C4, C5
22		C6, C4

V. Problem Formulation

The optimization model is formulated as mixed-integer linear programming problem using a routed-based representation on a directed graph as previously shown in Fig. 2. Prior to present the mathematical structure, some basic notation is introduced in the following.

A. Sets

- A^r Set of links contained in route *r*
- B Set of available bays
- B^k Set of available bays where truck *k* can park
- $B^{k1,k2}$ Set of available bays where trucks *k1*, *k2* can park
- I^k Set of customers to whom truck *k* can deliver product
- I^v Set of customers to whom company *v* delivers product
- K Set of available trucks
- K^v Set of available trucks for company *v*
- R^k Set of candidate routes for truck *k*
- R_b^k Set of candidate routes where truck *k* uses bay *b*
- R_i^k Set of candidate routes to deliver demanded product to customer *i* by truck *k*
- V Set of companies operating in the study area

B. Parameters

- d_i^v Product demand of customer *i* to company *v*
- st_{br}^k Service time from bay *b* to customers in route *r* using a dolly of truck *k*
- M Large constant to disable associated constraints
- q^k Capacity of truck *k*
- l_0^k Earliest departure time of truck *k* from its depot
- u_0^k Latest arrival time of truck *k* at its depot
- l_b^r Earliest arrival time at bay *b* when route *r* is used
- u_b^r Latest arrival time at bay *b* when route *r* is used
- t_a^k Travel time through link *a*
- w_i^v Income from customer *i* for company *v*
- w_b Set-up cost of bay *b*
- w^k Operation cost of truck *k* per unit of time

C. Decision variables

- h_r^k If route *r* is assigned to truck *k*
- x_i^v If customer *i* is not served by any truck of company *v*
- y_b If bay *b* is used by some truck
- $z_b^{k1,k2}$ If truck *k1* arrives before than truck *k2* to bay *b*
- D^k Departure time of truck *k* from its depot
- T_b^k Arrival time of truck *k* at bay *b*

D. Optimization model

$$\max_{\boldsymbol{h},\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{D},\boldsymbol{T}} \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}^k} w_i^k \sum_{r\in\mathcal{R}_i^k} h_r^k - \sum_{k\in\mathcal{K}} w^k\left(T_0^k - D^k\right) - \sum_{b\in\mathcal{B}} w_b y_b \tag{1}$$

$$s.t.:$$

$$\sum_{k\in\mathcal{K}^v}\sum_{r\in\mathcal{R}_i^k} h_r^k + x_i^v = 1, \quad \forall v\in\mathcal{V},\ i\in\mathcal{I}^v \tag{2}$$

$$\sum_{i\in\mathcal{I}^k} d_i^k \sum_{r\in\mathcal{R}_i^k} h_r^k \le q^k, \quad \forall k\in\mathcal{K} \tag{3}$$

$$\sum_{r\in\mathcal{R}^k} h_r^k \le 1, \quad \forall k\in\mathcal{K} \tag{4}$$

$$h_r^k \le y_b, \quad \forall k\in\mathcal{K},\ b\in\mathcal{B}^k,\ r\in\mathcal{R}_b^k \tag{5}$$

$$T_{j(a)}^k \ge D^k + t_a^k - M(1-h_r^k), \quad \forall k\in\mathcal{K},\ r\in\mathcal{R}^k,\ a\in\mathcal{A}^r : i(a)=0 \tag{6}$$

$$T_{j(a)}^k \ge T_{i(a)}^k + t_a^k + st_{i(a)r}^k - M(1-h_r^k), \quad \forall k\in\mathcal{K},\ r\in\mathcal{R}^k,\ a\in\mathcal{A}^r : i(a)\neq 0 \tag{7}$$

$$\sum_{r\in\mathcal{R}_b^k} l_b^r h_r^k \le T_b^k \le \sum_{r\in\mathcal{R}_b^k} u_b^r h_r^k, \quad \forall k\in\mathcal{K},\ b\in\mathcal{B}^k \tag{8}$$

$$\sum_{r\in\mathcal{R}_b^{k_1}} h_r^{k_1} + \sum_{r\in\mathcal{R}_b^{k_2}} h_r^{k_2} \le z_b^{k_1,k_2} + z_b^{k_2,k_1} + 1,\ \forall k_1,k_2\in\mathcal{K} : k_1\neq k_2,\ b\in\mathcal{B}^{k_1,k_2} \tag{9}$$

$$T_b^{k_2} \ge T_b^{k_1} + \sum_{r\in\mathcal{R}_b^{k_1}} st_{br}^{k_1} h_r^{k_1} - M(1-z_b^{k_1,k_2}),\ \forall k_1,k_2\in\mathcal{K} : k_1\neq k_2,\ b\in\mathcal{B}^{k_1,k_2} \tag{10}$$

$$D^k \ge l_0^k \sum_{r\in\mathcal{R}^k} h_r^k,\ \forall k\in\mathcal{K} \tag{11}$$

$$T_0^k \le u_0^k \sum_{r\in\mathcal{R}^k} h_r^k,\ \forall k\in\mathcal{K} \tag{12}$$

$$h_r^k \in \{0,1\},\ \forall k\in\mathcal{K},\ r\in\mathcal{R}^k \tag{13}$$

$$y_b \in \{0,1\},\ \forall b\in\mathcal{B} \tag{14}$$

$$x_i^v \in \{0,1\},\ \forall v\in\mathcal{V},\ i\in\mathcal{I}^v \tag{15}$$

$$z_b^{k_1,k_2} \in \{0,1\},\ \forall k_1,k_2\in\mathcal{K} : k_1\neq k_2,\ b\in\mathcal{B}^{k_1,k_2} \tag{16}$$

The objective function (1) maximizes the global benefit, considering all companies delivering product to customers in the study area. The benefit is computed according to the income from each product delivered to the customers (first term), the operation costs of the used trucks (second term), and the setup costs of the used bays (third term); while satisfying the following requirements.

Constraints (2) determine if some companies' trucks deliver the product to each of their customers. To this end, one of the two terms contained on the left side should be enabled (i.e: the subset of binary variables h_r^k denoting the routes *r* serving customer *i* by trucks of company v, or the binary variable x_i^v denoting if customer *i* has not been served by company *v*). Next constraints (3) ensure that the total amount of loaded product at each truck does not exceed its capacity. Constraints (4) verify that each truck is assigned to one route at most. Next constraints (5) ensure that set-up costs of the bays visited in the used routes are evaluated. Constraints (6)-(7) schedule the arrival times of trucks to each location contained in their used routes. Equations (6) schedule the arrival time of a truck to its first visited bay, considering the departure time from its depot and the travel time from the depot to the bay. Complementary, (7) schedule the arrival time of a truck either to subsequent bays or to its depot, considering the arrival and service times of the previously visited bay and the travel time between the pair either of consecutive visited bays or between the bay and the depot. The large constant *M* allows disabling the subset of these constraints associated with the unused truck routes. Next constraints (8) set the earliest and latest arrival times of trucks at bays depending on the used routes. These times correspond to the lower and upper limits of the customers' time windows to be delivered their demand from the bay on that route (see section III for further details). Constraints (9)-(10) coordinate the arrival times of two trucks *k1* and *k2* at the same bay. Equations (9) enable one of the binary variables that decide which truck parks first (i.e. $z_b^{k1,k2} = 1$ or $z_b^{k2,k1} = 1$) in case that both trucks use the same bay at some point during their routes. If so, (10) ensure that park times of the two trucks do not overlap. For instance, if $z_b^{k1,k2} = 1$ then (10) force truck *k2* to arrive having departed truck *k1* from the bay. Otherwise, if $z_b^{k2,k1} = 1$ then is the other way around. Next constraints (11)-(12) impose the earliest departure times and latest arrival times of the used trucks from/to their depots, respectively. Finally, constraints (13)-(16) impose the integrality on the binary variables.

VI. NUMERICAL EXPERIMENTS

Preliminary tests were conducted on the planning of fruits juice delivery in the historic center of Querétaro (México). Currently, the delivery company operates with only one truck in that area, and its depot is located on the periphery of the city as shown in Figs. 3-4. Daily, 147 customers demanding 117 boxes of fruits juice are served with that truck. The boxes must be delivered from 9:00h to 18:00h, and the route duration, starting and finishing at the company depot, cannot exceed 13 hours.

There exist 20 bays in which the truck can park and transfer the demanded product to the dolly. Having applied the clustering procedure described in section IV, an average of 1.5 customers' groups per bay, and a maximum of three were obtained. Using the customers' groups information, 10 candidate routes were identified.

Regarding to costs, 1.35 pesos per driven kilometer was used as operation cost for the truck route. No breaks due to rest or meals were considered. Set-up costs for used bays were also neglected. Finally, an income of 150 pesos was taken into account for each delivered box.

Within a few minutes, problem (1)-(16) was solved to optimality using the optimization platform of PTV-Group, and the resulting route was compared with the one planned by the company.

Table IV shows the main indicators of the planned routes, including the number of delivered boxes, the travel distance by truck, the total route duration, the service time using the dolly, and the empty time. The later considers the travel time by truck without load (i.e., when the truck has delivered all boxes and is then going back to its depot).

TABLE IV. COMPARISON OF THE PLANNED ROUTE BY THE COMPANY AND OURS

Scenario	Main Indicators					
	Delivered Boxes	***Travel Distance***	***Route Duration***	***Travel time***	***Service time***	***Empty time***
Company	177	106.6 km	9h 14 min	3h 50 min	5h 24 min	21 min
Our		46.1 km	6h 49 min	1h 37 min	5h 12 min	10 min

One can observe that our planned route significantly reduces the travel distance by truck, the total route duration, and the empty travel time by car, while delivering all boxes as well. However, the service time is quite similar in both planned routes and is much higher than the travel time by truck. One possible solution for this bottleneck problem is to use an urban third-party logistics provider, which takes the freight from the companies' trucks at some cross-docking center and delivers it to stores using a bicycle or a smaller truck.

Fig. 3-4 depict the traces of the company route and our planned route, respectively. Notice that customers in our planned route are only attended once; whereas, in the company route, customers are visited several times.

Fig. 3. Company route for the delivery of Fruits Juice.

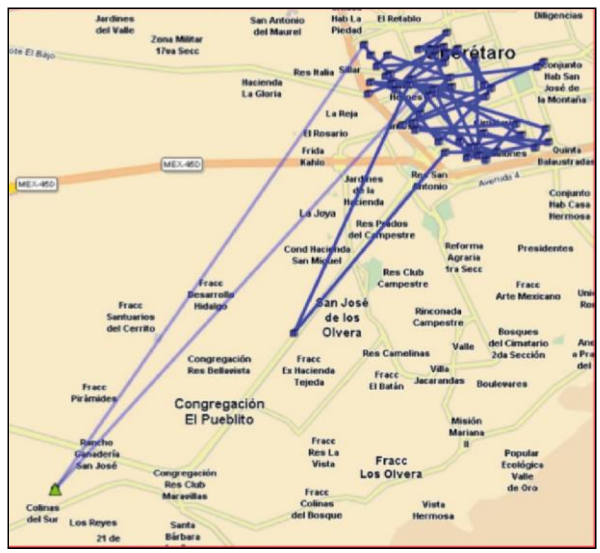
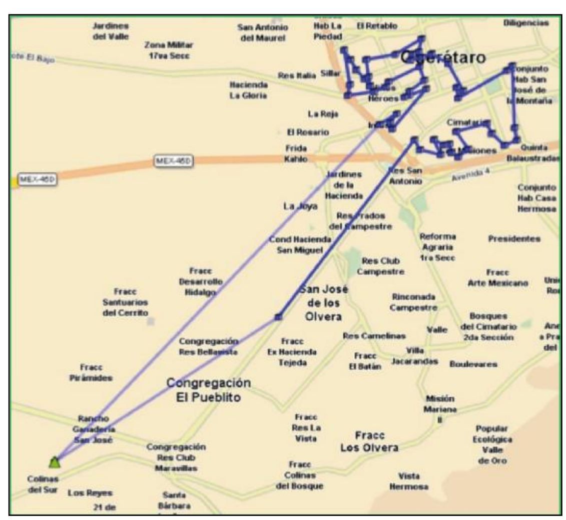


Fig. 4. Our proposed route for the delivy of Fruits Juice.



VII. Conclusions and Further research

This work presents an extension of the Truck and Trailer Routing Problem where several depots and customers with multiple time windows are considered. The inclusion of these two features possess a great difficulty when planning the routes of several companies because customers demand products from more than one company (located at a different depot) and, thus, an appropriate coordination among trucks serving those customers must be done.

The problem is formulated as a mixed-integer linear programming model with the aim of maximizing the total benefit. A route-based representation on a directed graph is used in order to efficiently solve real-sized instances. Customers' clustering and route enumeration methods are devised for defining the candidate routes to be evaluated by the model.

The daily delivery of fruits juice is planned with one truck. The results show that the optimal route is found within a few minutes, and that service times to customers using the dollies are three times higher than travel times of the trucks. Additionally, the travel distances by trucks are reduced from 106.6 km to 46.1 km. Since the pollutant gases emitted are directly proportional to the travel distance, the proposed approach is also environmentally friend.

Currently, the daily delivery of product from other three companies is being analyzed. It is planned to compute the scheduling of at least one route for each company altogether, in order to evaluate the difficulty in coordinating the product delivery from different companies to customers. When doing this, an algorithm to efficiently compute multiple routes against solving directly the model (1)-(16) is going to be devised.

Another important aspect is to ensure that the obtained solution satisfies the Nash equilibrium. This means that any player (company) can change its routes unilaterally. One possible solution is to set a combinatorial auction mechanism as proposed in [18]. In this auction, companies bid for using park places at compatible time slots. As a result, the authority determines which company and which price has to be paid for using the park places at desired time slots.

Finally, but not least, some environmental considerations can be explicitly considered in the objective function as a cost to be paid by companies. This cost is usually considered proportional to the CO_2 emissions that can be evaluated depending on some factors like the carried load and the travel speed of the truck.

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